Add support for differentiating functor objects in clad

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Add support for differentiating functor objects

What is meant by differentiating functors?

Differentiating functors means differentiating the call operator (\texttt{operator()}) member function defined by the functor type and executing the differentiated function using a reference to the functor object. Thus, the differentiated call operator have access to the member variables and member functions of the functor object. Despite the differences, differentiating functors is remarkably similar to differentiating functions in clad.

```
class ElectrostaticForce
{
    double q1, q2;
    const double k = 8.99e9;

    public:
    ElectrostaticForce(double p_q1, double p_q2): q1(p_q1), q2(p_q2)
    {
        double operator()(double radius)
        {
            return k * q1 * q2 / (radius + radius);
        }
    }
}
```

Differentiating functors example

Results:

- All clad differentiation functions now support differentiating functor objects.
- As a prerequisite for the implementation of differentiating functors, differentiation of member functions using clad reverse mode differentiation functions have been improved and support for differentiation of member function is added to clad::besselian.
- Just like functions, functors can also be passed both by pointers and by reference to the clad differentiation functions.
- Functions of template types and template specialization types are also supported.
- Differentiating functors using the forward mode AD (\texttt{clad::differentiate}), also supports differentiating with respect to member variables.

Add support for differentiating lambda expressions

Lambda expressions are C++ way of creating a closure, that is, an anonymous function object capable of capturing variables in scope.

```
auto momentum = [(double mass, double velocity)]
    { return mass * velocity; }
```

Differentiating lambda expressions example

Results:

- All clad differentiation functions now support differentiating lambda expressions.
- Just like functions and functors, lambda expressions can also be passed both by pointers and by reference to the clad differentiation functions.

Add support for differentiating more C++ syntax

Long term goal of clad is to support all C++ syntax. Support for more C++ syntax is added to bring clad one step closer to this goal.

Results:

- \texttt{while} and \texttt{do-while} statements are now supported in both the forward and the reverse mode automatic differentiation in clad.
- \texttt{switch}, \texttt{continue}, and \texttt{break} statements are now supported in the forward mode automatic differentiation in clad.

Automatic verification of the reverse mode AD

An assert-based testing framework, which can be optionally enabled using a compile-time flag, is added to verify derivatives produced by the reverse mode AD using the forward mode AD. The main goal of this is to make clad more robust by making it easier to find any inconsistencies and errors in the implementation of the forward and the reverse mode automatic differentiation.

The design of the automatic verification is very simple yet effective. It involves modifying the reverse mode gradient function to cross-verify the derivatives produced by the gradient function using the corresponding forward mode derivative functions.

Automatic verification modifies the gradient function as follows:

```
double fn_{\text{grad}}(\text{double } i, \text{ double } j)
{
    // copy make of all the arguments.
    // This will be used to call the forward mode differentiated functions.
    double \_p_\_i = i;
    double \_p_\_j = j;
    ... // usual code to calculate the function gradient.
    void cross-verify derivative at \_p_\_i and \_p_\_j
    {
        auto \_d_\_\_i = \_1_{\_\_\_grad}(\_p_\_i, \_p_\_j);
        auto \_d_\_\_j = \_2_{\_\_\_grad}(\_p_\_i, \_p_\_j);
        ...
    }
}
```

If verification fails, then the function prints an Assertion failed message and aborts the program.

Future work

- Add support for differentiating calls to member functions.
- Add support for differentiating functors with multiple overloaded call operators.
- Add automatic verification of the forward mode AD using the reverse mode AD.
- Add automatic verification of the automatic differentiation using numerical differentiation.

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