Optimizing automatic differentiation using activity analysis

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I’m from originally from Kyiv, Ukraine

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So what is activity analysis(AA)?

First a bit of motivation…

Sometimes Clad produces adjoints that are useless for the desired final derivative. Let's call those variables ***passive***. Otherwise, the variable is called ***active***. Now Clad assumes all variables are active, but we can do much better using AA.

Lets see the example:

code	forward mode	fm+aa

f(a, b, c):
  x = a*b
 	
  d = a*c
 	
  return x
	f_darg0(a, b, c):
	  d_a=1
	  d_b=0
	  d_c=0
	
	  d_x = d_a * b + a * d_b
	  x = a*b
	
	  d_d = d_a * c + a * d_c
	  d = a*c
	
	  return d_x
		f_darg0(a, b, c):
		  d_a=1
		  d_b=0
		
		  d_x = d_a * b + a * d_b
		  x = a*b
		
		  d = a*c
		
		  return d_x

AA is the combination of a forward and a backward analysis.

It propagates forward the **Varied** set of the variables that depend in a differentiable way on some independent input. Similarly, it propagates backwards the **Useful** set of the variables that influence some dependent output in a differentiable way.

Since the relation “depends in a differentiable way of” is transitive on code sequences, the essential equations of the propagation are:

$$Varied^{+}(I) \;=\; Varied^{-}(I) \;\times\; Diff - depp(I)$$

$$Useful^{-}(I) \;=\; Diff - dep(I) \;\times\; Useful^{+}(I)$$

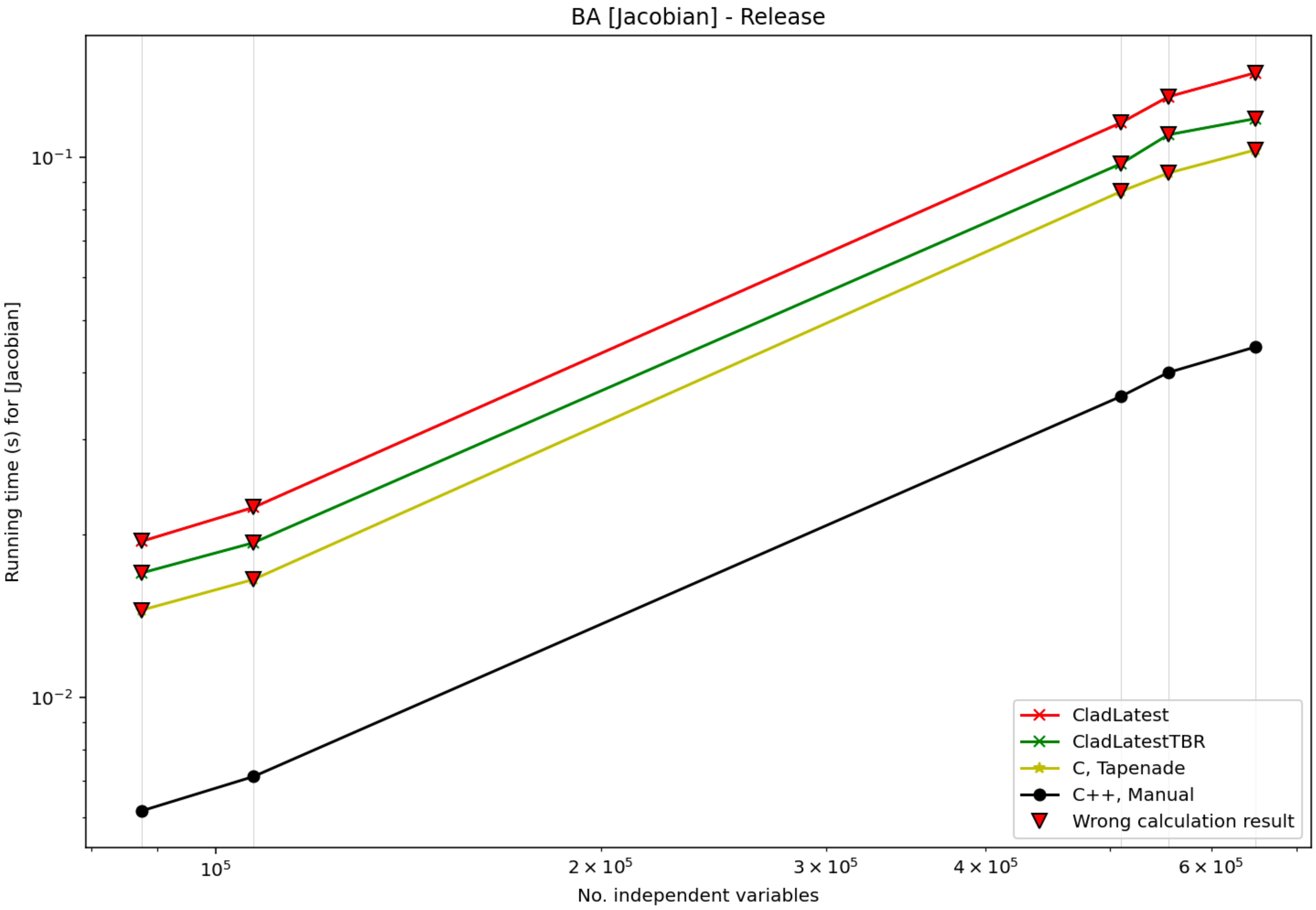
Where $Varied^{-}(I)$, $Varied^{+}(I)$ are sets of **Varied** variables before and after $I - th$ instruction,

$(v_1, v_2) \in Diff - dep(I)$ iff v_2 depends on v_1 after $I - th$ instruction,

$v_2 \;\in\; S \;\times\; Diff - dep(I) \Longleftrightarrow \exists v_1 \;\in\; S, (v_1, v_2) \;\in\; Diff - dep(I)$

And finally we define the set of all *active* variables as follows:

$$Active^{+}(I) \;=\; Varied^{+}(I) \;\cap\; Useful^{+}(I)$$


BA [Jacobian] - Release
Running time (s) for [Jacobian]
10^{-1}
10^{-2}
10^5
2×10^5
3×10^5
4×10^5
6×10^5
No. independent variables
CladLatest
CladLatestTBR
C, Tapenade
C++, Manual
Wrong calculation result

Note:

After AA is implemented and both AA and TBR analysis are default, there is a potential in modifying TBR using AA.

References

[1] L.Hascoët, V.Pascual. The Tapenade Automatic Differentiation Tool: Principles, Model, and Specification. *ACM Transactions on Mathematical Software 39(3):20:1-20:43.*